DESIGN CONCEPTS FOR LAMINATED COMPOSITE MATERIALS WITH THERMAL AND/OR MECHANICAL COUPLING RESPONSE

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1. Introduction

The main focus of this work is the identification of laminate configurations which possess complex mechanical couplings but remain hygro-thermally curvature-stable (HTCS) or warp-free.

HTCS laminates will allow a potentially broad range of exotic mechanical coupling attributes to be exploited without the complicating issue of thermal distortions, which are an inevitable consequence of the high temperature curing process.

Aero-elastic compliant rotor blades with tailored extension-twist coupling[*] is an example of one such laminate design that requires either specially curved tooling or hygro-thermally curvature-stable properties in order to remain flat (or possess the desired shape) after high temperature curing.

[*] Winckler, S. J. (1985) "Hygrothermally curvature stable laminates with tension-torsion coupling. *J. American Helicopter Society*, **31**: 56-58.

2. Characterisation of laminates

Laminated composite materials may be characterized in terms of their unique coupling behaviour in response to mechanical and/or thermal loading; coupling behaviour which is not present in conventional materials (metals).

The coupling behaviour, which is dependent on the form of the elements in each of the extensional (**A**), coupling (**B**) and bending (**D**) stiffness matrices may be described by a <u>response based labelling</u> system or <u>compact matrix notation</u>.

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} \mathrm{A}_{11} & \mathrm{A}_{12} & \mathrm{A}_{16} \\ & \mathrm{A}_{22} & \mathrm{A}_{26} \\ \mathrm{Sym.} & & \mathrm{A}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} \mathrm{B}_{11} & \mathrm{B}_{12} & \mathrm{B}_{16} \\ & \mathrm{B}_{22} & \mathrm{B}_{26} \\ \mathrm{Sym.} & & \mathrm{B}_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} \mathrm{B}_{11} & \mathrm{B}_{12} & \mathrm{B}_{16} \\ & \mathrm{B}_{22} & \mathrm{B}_{26} \\ \mathrm{Sym.} & & \mathrm{B}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} \mathrm{D}_{11} & \mathrm{D}_{12} & \mathrm{D}_{16} \\ & \mathrm{D}_{22} & \mathrm{D}_{26} \\ \mathrm{Sym.} & & \mathrm{D}_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

Table 1 – Subscript notation and response based labelling for: extensional stiffness matrix, **A**;

$\mathbf{A}_{\mathrm{S}}$	Simple laminate	$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$
$\mathbf{A}_{\mathrm{F}}$	Shear-Extension; *S-E*	$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{61} & A_{62} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$

Summary of Matrix sub-scripts

F = All elements Finite.

I = Fully Isotropic form; $A_{11} = A_{22}$; $A_{66} = (A_{11} - A_{12})/2$.

S = Specially orthotropic (uncoupled or Simple) form.

Response based labelling incorporates a *cause* and *effect* pairing, which is reversible, i.e. an extensional force resultant (N_x) gives rise to a shearing strain (γ_{xy}) in an *E-S* laminate.

Table 1 – Subscript notation and response based labelling for: bending stiffness matrix, **D**.

$\mathbf{D}_S$	Simple laminate	$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$
$\mathbf{D}_F$	Twisting-Bending; *T-B*	$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{21} & D_{22} & D_{26} \\ D_{61} & D_{62} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$

Summary of Matrix sub-scripts

F = All elements Finite.
I = Fully Isotropic form; $D_{11} = D_{22}$; $D_{66} = (D_{11} - D_{12})/2$.
S = Specially orthotropic (uncoupled or Simple) form.

Table 1 – Subscript notation and response based labelling for: coupling stiffness matrix, **B**;

$\mathbf{B}_0$	Uncoupled	$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$	$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$
$\mathbf{B}_l$	Extension-Bending; <u>*E-B*</u>	$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & 0 & 0 \\ 0 & B_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$	$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & 0 & 0 \\ 0 & B_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$

Continued

Summary of Matrix sub-scripts
0 = All elements (of stiffness matrix) zero.
l = Leading diagonal elements (B_{11}, $B_{22} \neq 0$) of **B** matrix non-zero, all other elements zero.

The *cause* and *effect* coupling relationship also corresponds to an applied (bending and/or twisting) moment resultant and the associated extensional (and/or shearing) strains, in this case a <u>*B-E*</u> laminate.

Table 1 – Subscript notation and response based labelling for: coupling stiffness matrix, **B**;

Continued

$\mathbf{B}_t$	Extension-Twisting and Shearing-Bending; <u>*E-T*</u>-<u>*S-B*</u>	$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & \mathrm{B}_{16} \\ 0 & 0 & \mathrm{B}_{26} \\ \mathrm{B}_{61} & \mathrm{B}_{62} & 0 \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$	$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & \mathrm{B}_{16} \\ 0 & 0 & \mathrm{B}_{26} \\ \mathrm{B}_{61} & \mathrm{B}_{62} & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$
$\mathbf{B}_{lt}$	Extension-Bending, Extension-Twisting and Shearing-Bending; <u>*E-B*</u>-<u>*E-T*</u>-<u>*S-B*</u>	$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} \mathrm{B}_{11} & 0 & \mathrm{B}_{16} \\ 0 & \mathrm{B}_{22} & \mathrm{B}_{26} \\ \mathrm{B}_{61} & \mathrm{B}_{62} & 0 \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$	$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} \mathrm{B}_{11} & 0 & \mathrm{B}_{16} \\ 0 & \mathrm{B}_{22} & \mathrm{B}_{26} \\ \mathrm{B}_{61} & \mathrm{B}_{62} & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$

Continued

Summary of Matrix sub-scripts

l	=	Leading diagonal elements (B_{11}, $\mathrm{B}_{22} \neq 0$) of **B** matrix non-zero, all other elements zero.
t	=	Off-diagonal elements (B_{16}, $\mathrm{B}_{26} \neq 0$) of **B** matrix non-zero, all other elements zero.

Table 1 – Subscript notation and response based labelling for: coupling stiffness matrix, **B**;

Continued

$\mathbf{B}_{\mathrm{S}}$	Extension-Bending and Shearing-Twisting; <u>*E*</u>*-*<u>*B*</u>-<u>*S*</u>*-*<u>*T*</u>	$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} \mathrm{B}_{11} & \mathrm{B}_{12} & 0 \\ \mathrm{B}_{21} & \mathrm{B}_{22} & 0 \\ 0 & 0 & \mathrm{B}_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$	$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} \mathrm{B}_{11} & \mathrm{B}_{12} & 0 \\ \mathrm{B}_{21} & \mathrm{B}_{22} & 0 \\ 0 & 0 & \mathrm{B}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$
$\mathbf{B}_{\mathrm{F}}$	Extension-Bending, Shearing-Bending, Extension-Twisting, and Shearing-Twisting; <u>*E*</u>*-*<u>*B*</u>-<u>*S*</u>*-*<u>*B*</u>-<u>*E*</u>*-*<u>*T*</u>-<u>*S*</u>*-*<u>*T*</u>	$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} \mathrm{B}_{11} & \mathrm{B}_{12} & \mathrm{B}_{16} \\ \mathrm{B}_{21} & \mathrm{B}_{22} & \mathrm{B}_{26} \\ \mathrm{B}_{61} & \mathrm{B}_{62} & \mathrm{B}_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$	$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} \mathrm{B}_{11} & \mathrm{B}_{12} & \mathrm{B}_{16} \\ \mathrm{B}_{21} & \mathrm{B}_{22} & \mathrm{B}_{26} \\ \mathrm{B}_{61} & \mathrm{B}_{62} & \mathrm{B}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$

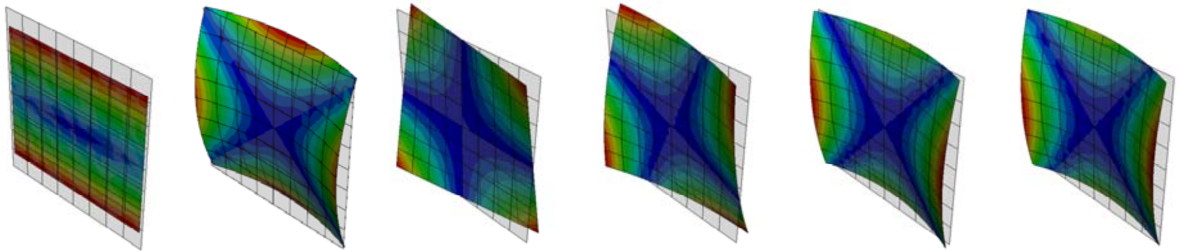
Concluded

Summary of Matrix sub-scripts

F = All elements <u>F</u>inite.

S = <u>S</u>pecially orthotropic (uncoupled or <u>S</u>imple) form; $\mathrm{B}_{16} = \mathrm{B}_{26} = 0$ when applied to **B** matrix.

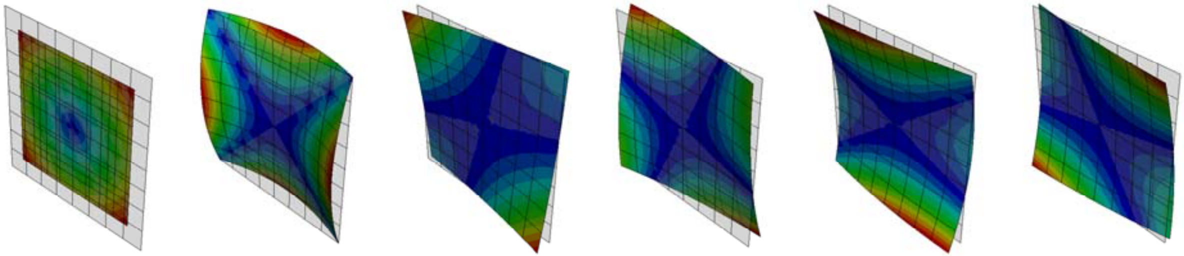
$\mathbf{A}_S\mathbf{B}_0\mathbf{D}_S$	$\mathbf{A}_S\mathbf{B}_l\mathbf{D}_S$	$\mathbf{A}_S\mathbf{B}_t\mathbf{D}_S$	$\mathbf{A}_S\mathbf{B}_{lt}\mathbf{D}_S$	$\mathbf{A}_S\mathbf{B}_S\mathbf{D}_S$	$\mathbf{A}_S\mathbf{B}_F\mathbf{D}_S$
$[+/-_2/\bigcirc/+_2/-]_T$	$[\pm/\bigcirc/-/\bigcirc/+/\bullet/\pm]_T$	$[+/-]_T$	$[+/\bigcirc/\bullet/-]_T$	$[+/-_2/\bigcirc/+_2/-/\bullet]_T$	$[+/-_2/+/\bigcirc/+/-/\bullet]_T$



$$\begin{bmatrix} B_{11} & 0 & 0 \\ 0 & B_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & B_{16} \\ 0 & 0 & B_{26} \\ B_{61} & B_{62} & 0 \end{bmatrix} \begin{bmatrix} B_{11} & 0 & B_{16} \\ 0 & B_{22} & B_{26} \\ B_{61} & B_{62} & 0 \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{21} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{61} & B_{62} & B_{66} \end{bmatrix}$$

Fig. 1 – Isolated coupling responses, due to free thermal contraction, for the: ($\mathbf{A}_S\mathbf{B}_0\mathbf{D}_S$) *Simple* or uncoupled laminate; ($\mathbf{A}_S\mathbf{B}_l\mathbf{D}_S$) *B-E* laminate with bending-extension coupling; ($\mathbf{A}_S\mathbf{B}_t\mathbf{D}_S$) *B-S-T-E* laminate with bending-shearing and twisting-extension coupling; ($\mathbf{A}_S\mathbf{B}_{lt}\mathbf{D}_S$) *B-E-B-S-T-E* laminate with bending-extension, bending-shearing and twisting-extension coupling; ($\mathbf{A}_S\mathbf{B}_S\mathbf{D}_S$) *B-E-T-S* laminate with bending-extension and twisting-shearing coupling and; ($\mathbf{A}_S\mathbf{B}_F\mathbf{D}_S$) *B-E-B-S-T-E-T-S* or fully coupled.

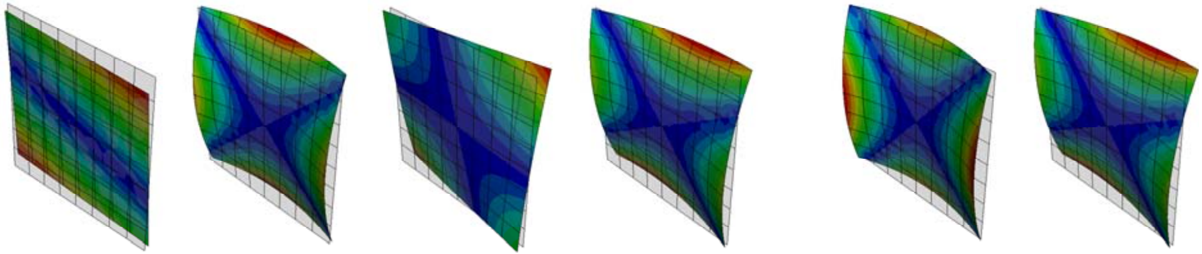
$\mathbf{A}_S\mathbf{B}_0\mathbf{D}_F$	$\mathbf{A}_S\mathbf{B}_l\mathbf{D}_F$	$\mathbf{A}_S\mathbf{B}_t\mathbf{D}_F$	$\mathbf{A}_S\mathbf{B}_{lt}\mathbf{D}_F$	$\mathbf{A}_S\mathbf{B}_S\mathbf{D}_F$	$\mathbf{A}_S\mathbf{B}_F\mathbf{D}_F$
$[+/-/-/+]_T$	$[+/\bigcirc/-_2/\bullet/+]_T$	$[+/-_3/+_2]_T$	$[+/\bigcirc_2/+/-_2/\bullet]_T$	$[+/-_2/+/\bigcirc]_T$	$[+/-/\bigcirc]_T$



$$\begin{bmatrix} B_{11} & 0 & 0 \\ 0 & B_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & B_{16} \\ 0 & 0 & B_{26} \\ B_{61} & B_{62} & 0 \end{bmatrix} \begin{bmatrix} B_{11} & 0 & B_{16} \\ 0 & B_{22} & B_{26} \\ B_{61} & B_{62} & 0 \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{21} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{61} & B_{62} & B_{66} \end{bmatrix}$$

Fig. 2 – Coupling responses, due to free thermal contraction, for ($\mathbf{A}_S\mathbf{B}_0\mathbf{D}_F$) <u>*B-T*</u> laminates with bending-twisting coupling combined with: ($\mathbf{A}_S\mathbf{B}_l\mathbf{D}_F$) <u>*B-E*</u> or bending-extension coupling; ($\mathbf{A}_S\mathbf{B}_t\mathbf{D}_F$) <u>*B-S*</u>-<u>*T-E*</u> or bending-shearing and twisting-extension coupling; ($\mathbf{A}_S\mathbf{B}_{lt}\mathbf{D}_F$) <u>*B-E*</u>-<u>*B-S*</u>-<u>*T-E*</u> or bending-extension, bending-shearing and twisting-extension coupling; ($\mathbf{A}_S\mathbf{B}_S\mathbf{D}_F$) <u>*B-E*</u>-<u>*T-S*</u> or bending-extension and twisting-shearing coupling and; ($\mathbf{A}_S\mathbf{B}_F\mathbf{D}_F$) <u>*B-E*</u>-<u>*B-S*</u>-<u>*T-E*</u>-<u>*T-S*</u> or fully coupled.

$\mathbf{A}_F\mathbf{B}_0\mathbf{D}_S$	$\mathbf{A}_F\mathbf{B}_l\mathbf{D}_S$	$\mathbf{A}_F\mathbf{B}_t\mathbf{D}_S$	$\mathbf{A}_F\mathbf{B}_{lt}\mathbf{D}_S$	$\mathbf{A}_F\mathbf{B}_S\mathbf{D}_S$	$\mathbf{A}_F\mathbf{B}_F\mathbf{D}_S$
[±/○/−/○/$-_3$/○/$-_3$/○/+]$_T$	[±/○/−/○/$-_3$/○/$-_3$/●/+]$_T$	[±/○/$-_3$/○/−/○/$-_3$/±/○/+]$_T$	[±/○/$-_3$/○/−/○/$-_3$/±/●/+]$_T$	[±/○/$-_7$/ +/●]$_T$	[+/$-_6$/○/$-_2$/●/+]$_T$



$$\begin{bmatrix} B_{11} & 0 & 0 \\ 0 & B_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & B_{16} \\ 0 & 0 & B_{26} \\ B_{61} & B_{62} & 0 \end{bmatrix} \begin{bmatrix} B_{11} & 0 & B_{16} \\ 0 & B_{22} & B_{26} \\ B_{61} & B_{62} & 0 \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{21} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{61} & B_{62} & B_{66} \end{bmatrix}$$

Fig. 4 – Coupling responses, due to free thermal contraction, for ($\mathbf{A}_F\mathbf{B}_0\mathbf{D}_S$) *E-S* laminates with extension-shearing coupling combined with: ($\mathbf{A}_F\mathbf{B}_l\mathbf{D}_S$) *B-E* or bending-extension coupling; ($\mathbf{A}_F\mathbf{B}_t\mathbf{D}_S$) *B-S*-*T-E* or bending-shearing and twisting-extension coupling; ($\mathbf{A}_F\mathbf{B}_{lt}\mathbf{D}_S$) *B-E*-*B-S*-*T-E* or bending-extension, bending-shearing and twisting-extension coupling; ($\mathbf{A}_F\mathbf{B}_S\mathbf{D}_S$) *B-E*-*T-S* or bending-extension and twisting-shearing coupling and; ($\mathbf{A}_F\mathbf{B}_F\mathbf{D}_S$) *B-E*-*B-S*-*T-E*-*T-S* or fully coupled.

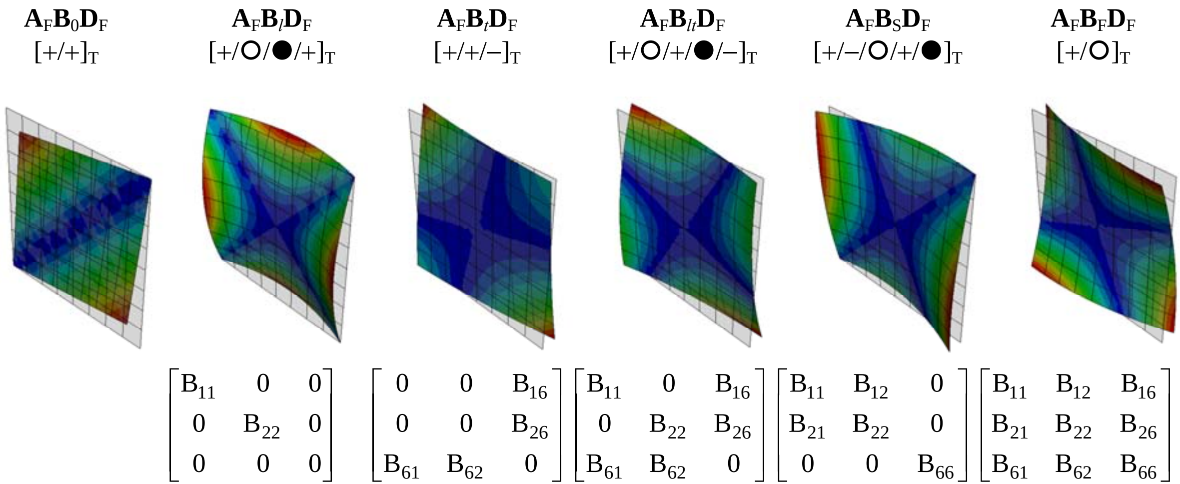


Fig. 3 – Coupling responses, due to free thermal contraction, for ($\mathbf{A}_F\mathbf{B}_0\mathbf{D}_F$) *E-S*;*B-T* laminates with extension-shearing and bending-twisting coupling combined with: ($\mathbf{A}_F\mathbf{B}_l\mathbf{D}_F$) *B-E* or bending-extension coupling; ($\mathbf{A}_F\mathbf{B}_t\mathbf{D}_F$) *B-S*-*T-E* or bending-shearing and twisting-extension coupling; ($\mathbf{A}_F\mathbf{B}_{lt}\mathbf{D}_F$) *B-E*-*B-S*-*T-E* or bending-extension, bending-shearing and twisting-extension coupling; ($\mathbf{A}_F\mathbf{B}_S\mathbf{D}_F$) *B-E*-*T-S* or bending-extension and twisting-shearing coupling and; ($\mathbf{A}_F\mathbf{B}_F\mathbf{D}_F$) *B-E*-*B-S*-*T-E*-*T-S* or fully coupled.

3. Laminate design

Lamination parameters offer a set of non-dimensional expressions when ply angles are a design constraint and were used in the algorithm for developing the solutions that follow.

3.1. *Equivalent Fully Isotropic Laminates*

These are used to normalise the stiffness properties that follow.

3.2. *Hygro-thermally Curvature-Stable or Warp-Free Laminates*

The manufacture of any of the foregoing coupled classes of laminate presents a particular challenge if mechanical coupling attributes are required without the thermal distortions illustrated; such distortions are a consequence of the high temperature curing process requirements.

Hygro-thermally curvature-stable or thermally warp-free laminates offer a tailored design solution.

3.2.1. *<u>E-B</u>-<u>S-T</u> coupled quasi-homogenous orthotropic laminates* ($\mathbf{A}_S\mathbf{B}_S\mathbf{D}_S \rightarrow \mathbf{A}_I\mathbf{B}_S\mathbf{D}_I$)

Quasi-homogenous orthotropic laminates possess concomitant extensional and bending stiffness properties, i.e. $D_{ij} = A_{ij}H^2/12$,

Eight hygro-thermally curvature-stable solutions† were found to exist in the range up to 21 plies when standard ply angles are adopted.

Table 2 – Hygro-thermally curvature-stable 16-ply quasi-homogeneous orthotropic stacking sequence configurations, together with the corresponding lamination parameter, ξ_6, representing $\mathbf{A}_I\mathbf{B}_S\mathbf{D}_I$ laminates with standard ply orientations ±45, 0 and 90° in place of symbols +, –, ○ and ●, respectively.

Stacking sequence	ξ_6
$[+/-/-/+/-/+/+/-/○/●/●/○/●/○/○/●]_T \equiv [+/-/-/+/-/+/+/-/●/○/○/●/○/●/●/○]_T$	1.00
$[+/-/-/+/○/●/●/○/-/+/+/-/●/○/○/●]_T \equiv [+/-/-/+/●/○/○/●/-/+/+/-/○/●/●/○]_T$	0.50
$[+/-/○/●/-/+/●/○/-/+/●/○/+/-/○/●]_T \equiv [+/-/●/○/-/+/○/●/-/+/○/●/+/-/●/○]_T$	0.25
$[+/○/-/●/-/●/+/○/-/●/+/○/+/○/-/●]_T \equiv [+/●/-/○/-/○/+/●/-/○/+/●/+/●/-/○]_T$	0.13

† York C. B. (2010b). Coupled Quasi-Homogeneous Orthotropic Laminates. *Proc. 16th International Conference on Mechanics of Composite Materials,* Riga, Latvia.

Table 3 – Conditions for hygro-thermally curvature-stable behaviour in coupled quasi-homogeneous orthotropic laminates.

Lamination parameters and stiffness relationships with respect to material axis alignment, β.		
$\beta = m\pi/2$	$\beta = \pi/8 + m\pi/4$ $(m = 0, 1, 2, 3)$	$\beta \neq m\pi/2, \pi/8 + m\pi/4$
	$\begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{11} & 0 \\ 0 & 0 & (A_{11}-A_{12})/2 \end{bmatrix}$ $\xi_1 = \xi_2 = \xi_3 = \xi_4 = 0$	
$\begin{bmatrix} B_{11} & -B_{11} & 0 \\ -B_{11} & B_{11} & 0 \\ 0 & 0 & -B_{11} \end{bmatrix}$ $\xi_5 = \xi_7 = \xi_8 = 0$	$\begin{bmatrix} 0 & 0 & B_{16} \\ 0 & 0 & -B_{16} \\ B_{16} & -B_{16} & 0 \end{bmatrix}$ $\xi_5 = \xi_6 = \xi_7 = 0$	$\begin{bmatrix} B_{11} & -B_{11} & B_{16} \\ -B_{11} & B_{11} & -B_{16} \\ B_{16} & -B_{16} & -B_{11} \end{bmatrix}$ $\xi_5 = \xi_7 = 0$
$\mathbf{A}_\mathrm{I}\mathbf{B}_\mathrm{S}\mathbf{D}_\mathrm{I}$	$\mathbf{A}_\mathrm{I}\mathbf{B}_t\mathbf{D}_\mathrm{I}$	$\mathbf{A}_\mathrm{I}\mathbf{B}_\mathrm{F}\mathbf{D}_\mathrm{I}$

3.2.2. *E-B-S-T coupled extensionally isotropic laminates* ($\mathbf{A_I B_S D_S}$)

$D_{ij} \neq A_{ij}H^2/12$ increases the design space to 8, 264 and 17,118 sequences with 12, 16 and 20 plies, respectively.

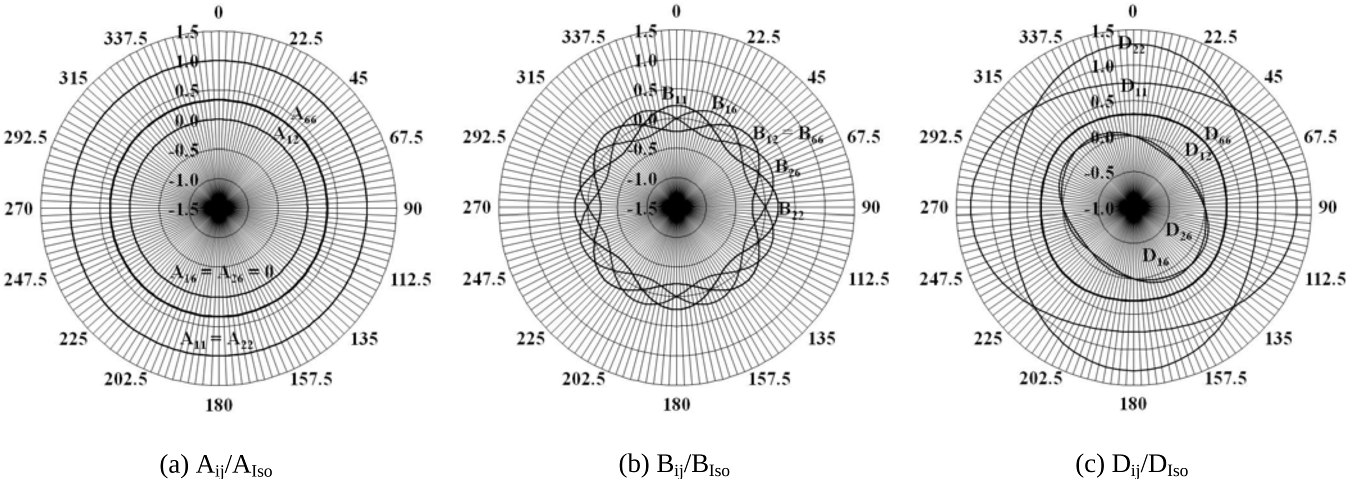
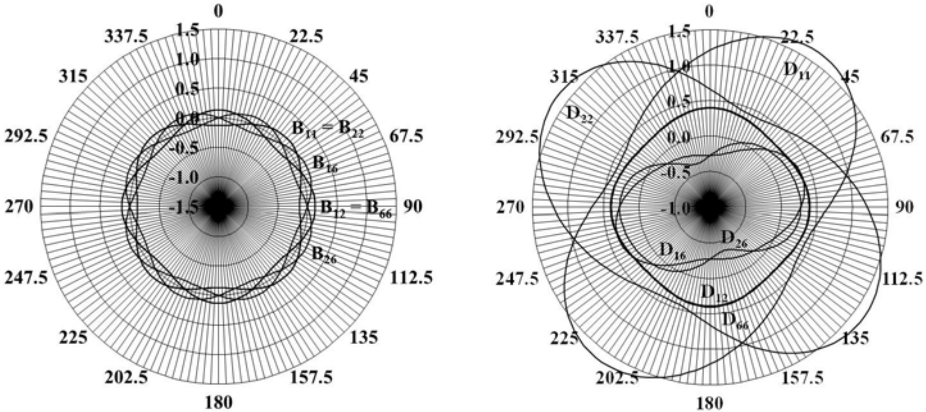


Figure 1 – Polar plots of the: (a) **A**; (b) **B** and (c) **D** matrices corresponding to off-axis material alignment, 0° ≤ β ≤ 360°, for the 12-ply $\mathbf{A_I B_S D_S}$ hygro-thermally curvature-stable laminate stacking sequence $[-/+_2/\bigcirc/-_2/+/\bullet_3/\bigcirc_2]_T$, assuming standard ply orientations ±45, 0 and 90° in place of symbols +, –, ○ and ●, respectively.

3.2.3. *E-B-S-T;B-T coupled extensionally isotropic laminates* ($\mathbf{A}_I\mathbf{B}_S\mathbf{D}_F$)

For D_{16}, $D_{26} \neq 0$, the design space contains 6, 280, 23,652 and 2,379,722 sequences with 8, 12, 16 and 20 plies.



(a) B_{ij}/B_{Iso} (b) D_{ij}/D_{Iso}

Figure 7 – Polar plots of the: (a) **B** and (b) **D** matrices corresponding to off-axis material alignment, 0° ≤ β ≤ 360°, for the 12-ply $\mathbf{A}_I\mathbf{B}_S\mathbf{D}_F$ hygro-thermally curvature-stable laminate stacking sequence $[-/\bullet_2/\circ_3/+_3/\bullet/-_2]_T$, assuming standard ply orientations ±45, 0 and 90° in place of symbols +, –, ○ and ●, respectively.

Chen[‡] demonstrated that the constraints on hygro-thermally curvature-stable laminates may be relaxed in comparison to those stated in Table 3, i.e.:

$$\xi_1 = \xi_3 = 0$$

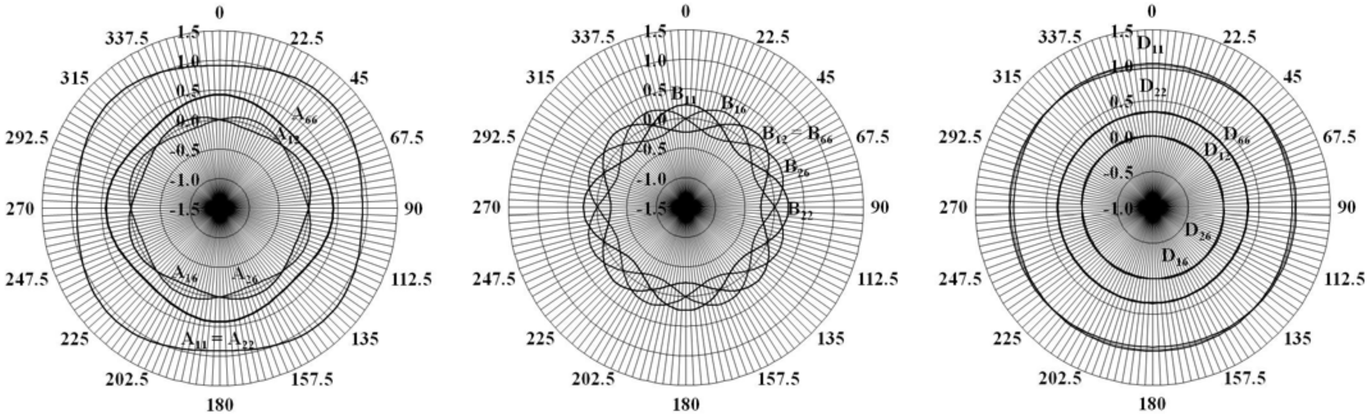
and

$$\begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{11} & -A_{16} \\ A_{16} & -A_{16} & A_{66} \end{bmatrix}$$

[‡] Chen, H. P. (2003). Study of hygrothermal isotropic layup and hygrothermal curvature-stable coupling composite laminates, *Proc. 44th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conf.*, Paper No. AIAA-2003-1506.

3.2.4. *E-B-S-T coupled laminates* ($\mathbf{A}_S\mathbf{B}_S\mathbf{D}_S$)

For A_{16}, $A_{26} \neq 0$, the design space contains 6, 524, and 35,610 solutions for 12, 16 and 20 ply laminates.



(a) A_{ij}/A_{Iso} (b) B_{ij}/B_{Iso} (c) D_{ij}/D_{Iso}

Figure 2 – Polar plots of the: (a) **A**; (b) **B** and (c) **D** matrices, corresponding to off-axis material alignment, $0° \leq \beta \leq 360°$, for the 12-ply $\mathbf{A}_S\mathbf{B}_S\mathbf{D}_S$ hygro-thermally curvature-stable laminate stacking sequence [–/+/+/–/+/–/–/+/●/○/○/●]$_T$, assuming standard ply orientations ±45, 0 and 90° in place of symbols +, –, ○ and ●, respectively.

Despite the presence of bending-twisting coupling behaviour for off-axis alignment, the properties of this laminate closely approximate isotropic behaviour in bending.

This is of particular interest in the context of aero-elastic compliant wind-turbine blade design, for passive load alleviation, where laminate level extension-shearing, extension-twisting (and shearing-bending) coupling provides the necessary and sufficient response to achieve bending-twisting and extension-twisting at the structural level, from aerodynamic and centripetal forces, respectively.

3.2.5. *E-B-S-T;B-T coupled laminates* ($\mathbf{A}_S\mathbf{B}_S\mathbf{D}_F$)

For A_{16}, $A_{26} \neq 0$ and D_{16}, $D_{26} \neq 0$, the design space contains 410, 40,808 and 4,515,473 solutions with 12, 16 and 20 plies.

4. Conclusions

Stacking sequence configurations for hygro-thermally curvature-stable laminates have been identified in **9** of the **24** classes of coupled laminate with standard ply angle orientations +45, –45, 0 and 90°.

All arise from the judicious re-alignment of the principal material axis of $\mathbf{A}_S\mathbf{B}_S\mathbf{D}_S$ or *B-E*-*T-S* laminates, or additionally $\mathbf{A}_S\mathbf{B}_S\mathbf{D}_F$ or *B-E*-*T-S*;*B-T* laminates.

Off-axis material alignments of these parent laminates gives rise to more complex combinations of coupled behaviour.

Extending the study to the consideration of double-angle-ply laminates, with θ, $-\theta$, $(\theta+90)$, $(-\theta+90)$ in place of the standard ply angle orientations, reveals additional classes of coupled laminate....